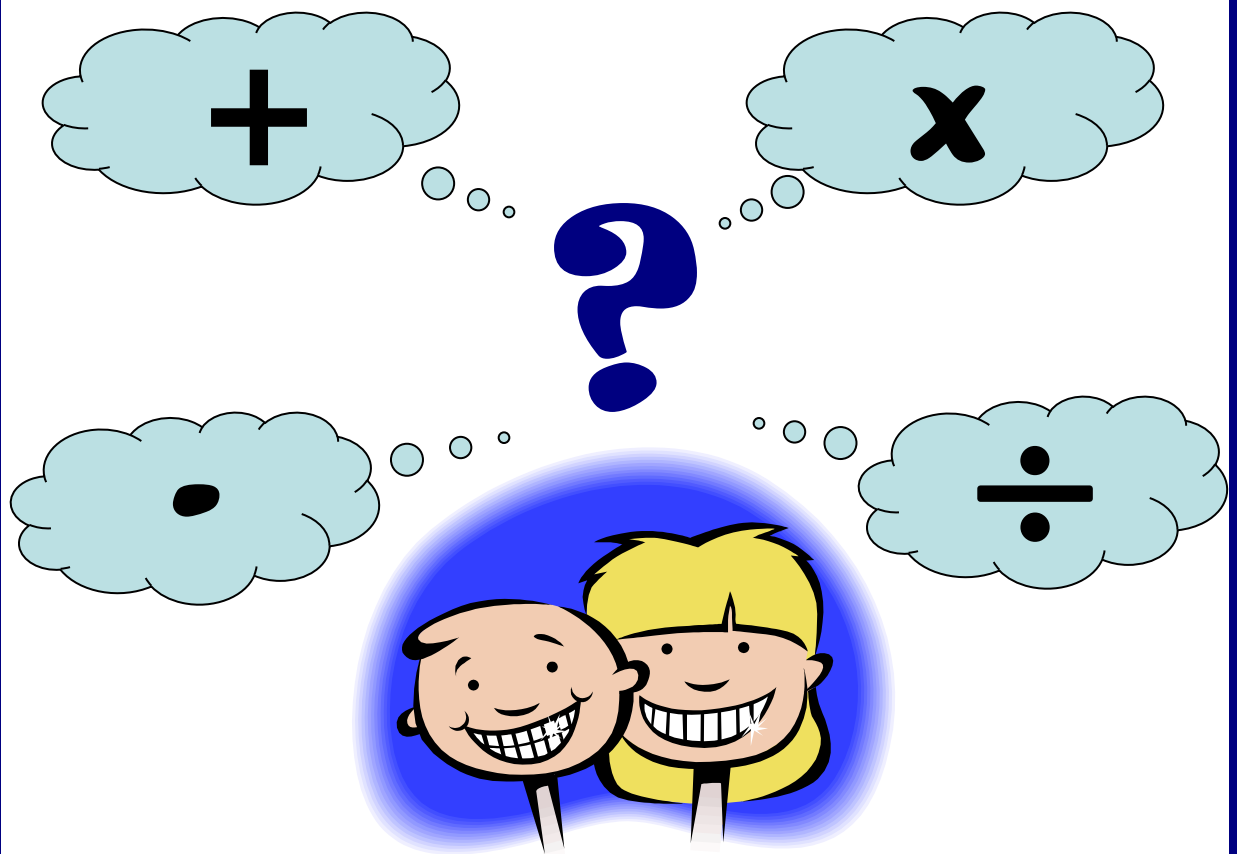


Progression in Calculations: A guide for parents



Darley Dene
Primary School

Introduction

Written methods of calculations are based on mental strategies. Each of the four operations builds on mental skills which provide the foundation for jottings and informal written methods of recording. Skills need to be taught, practised and reviewed constantly. These skills lead on to more formal written methods of calculation.

Strategies for calculation need to be supported by familiar models and images to reinforce understanding. When teaching a new strategy it is important to start with numbers that the child can easily manipulate so that they can understand the concept.

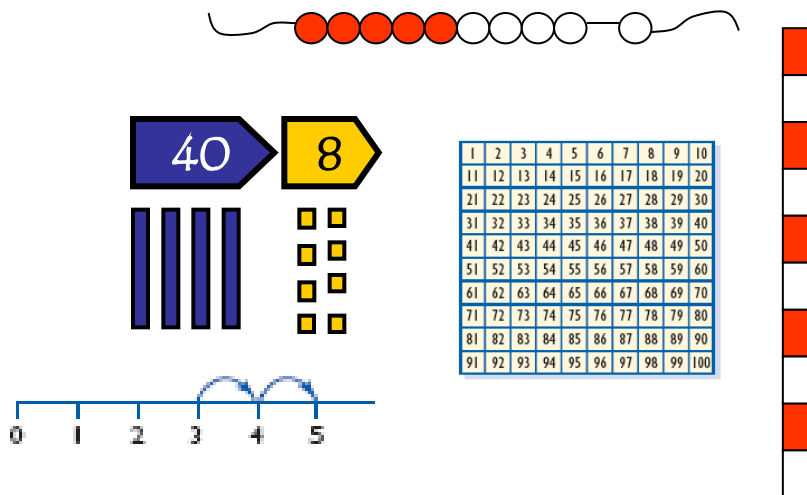
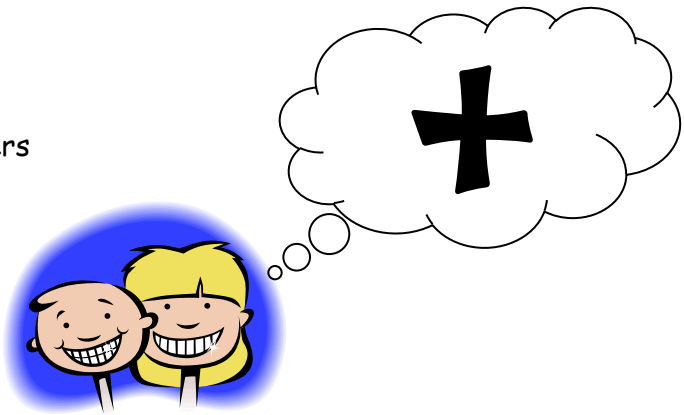
The transition between stages should not be hurried as not all children will be ready to move on to the next stage at the same time, therefore the progression in this document is outlined in stages. An indication as to which stage each year group begins their focus is also given; however previous stages may need to be revisited to consolidate understanding when introducing a new strategy.

A sound understanding of the number system is essential for children to carry out calculations efficiently and accurately.

Progression in Teaching Addition

Mental Skills

Recognise the size and position of numbers
Count on in ones and tens
Know number bonds to 10 and 20
Add multiples of 10 to any number
Partition and recombine numbers
Bridge through 10



Key Vocabulary

add
addition
plus
and
count on
more
sum
total
altogether
increase

add and count on
addition plus
more sum total
altogether increase

Recognise numbers 0 to 10

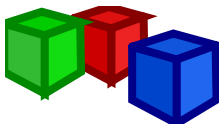
0 1 2 3 4 5 6 7 8 9 10



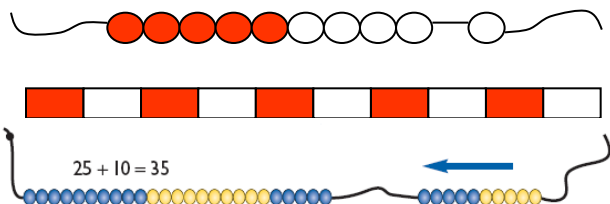
1, 2, 3, 4, 5, 6
... there are 6
teddies

Count reliably up to 10 everyday objects

Find one more than a number



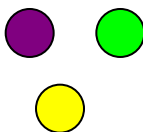
One more than
three is four



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Count in ones and tens

Begin to relate addition to
combining two groups of objects



and makes 5

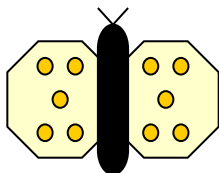
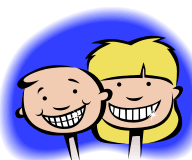
$$3 + 2 = 5$$



Count along a number line to
add numbers together

Begin to use the + and = signs to record
mental calculations in a number sentence

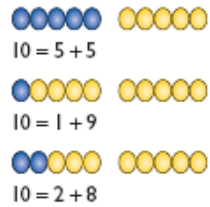
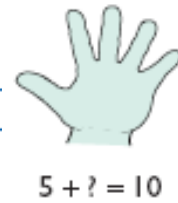
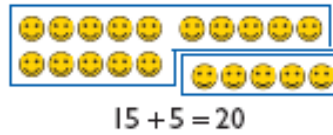
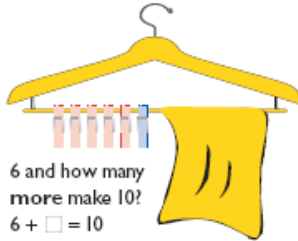
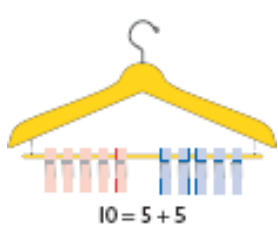
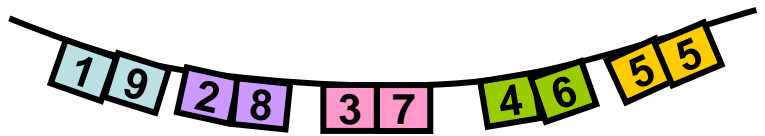
$$6 + 4 = 10$$



$$5 + 5 = 10$$

Know doubles of numbers

Know by heart all pairs of numbers with a total of 10 and 20



$$1 + 2 = 3$$



$$2 + 1 = 3$$



$$2 + 5 = 7$$

2 count on 5



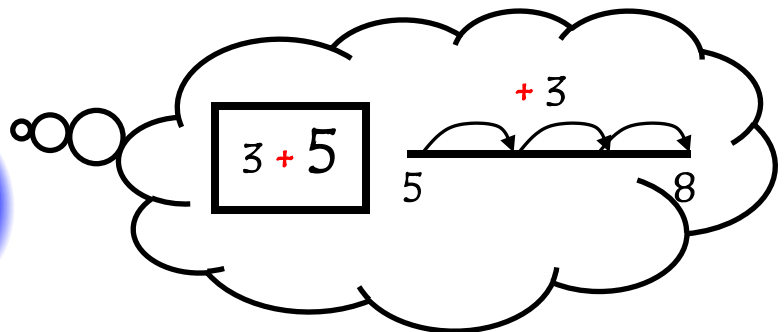
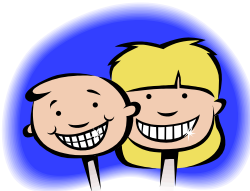
$$5 + 2 = 7$$

5 count on 2

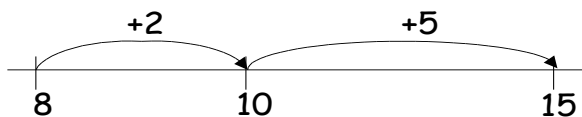


Know that addition can be done in any order

Put the biggest number first and count on

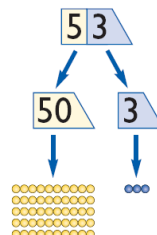


$$8 + 7 = 15$$



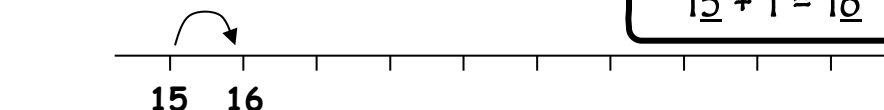
Add two single-digit numbers that bridge 10

Begin to partition numbers in order to add

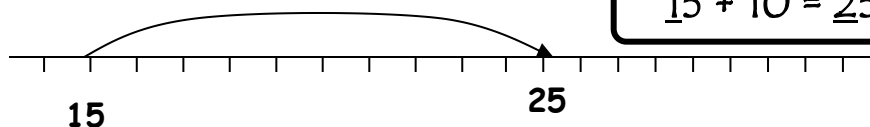


$$30p + 4p = 34p$$

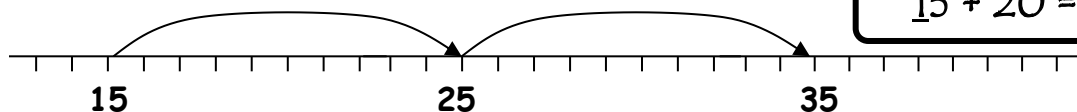
Know which digit changes when adding 1s or 10s to any number



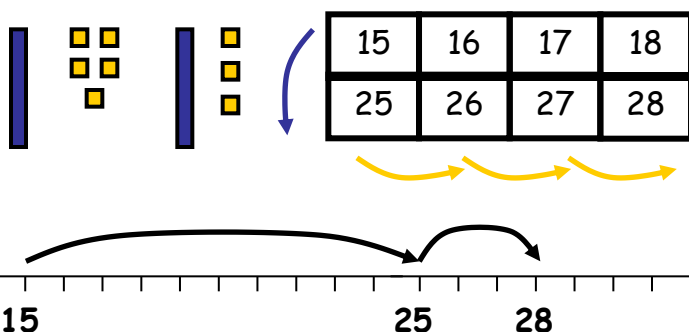
$$15 + 1 = 16$$



$$15 + 10 = 25$$



$$15 + 20 = 35$$



Adding two two-digit numbers (without bridging)

Counting in tens and ones
Partitioning and recombining

$$15 + 13 = 28$$

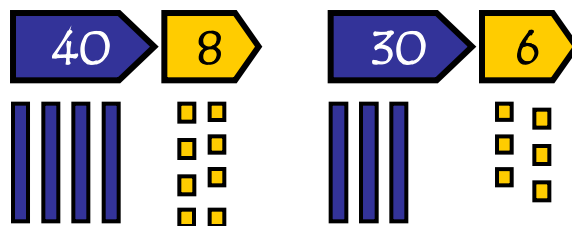
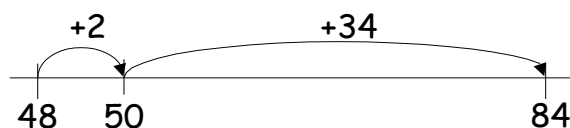
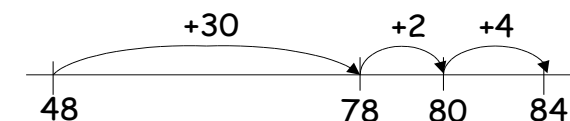
Adding two two-digit numbers (bridging through tens boundary)

Using a number line

OR

Using place value cards and place value apparatus to partition numbers and recombine

$$48 + 36 = 84$$



$$40 + 30 + 8 + 6$$

$$8 + 6 = 14$$

$$40 + 30 = 70$$

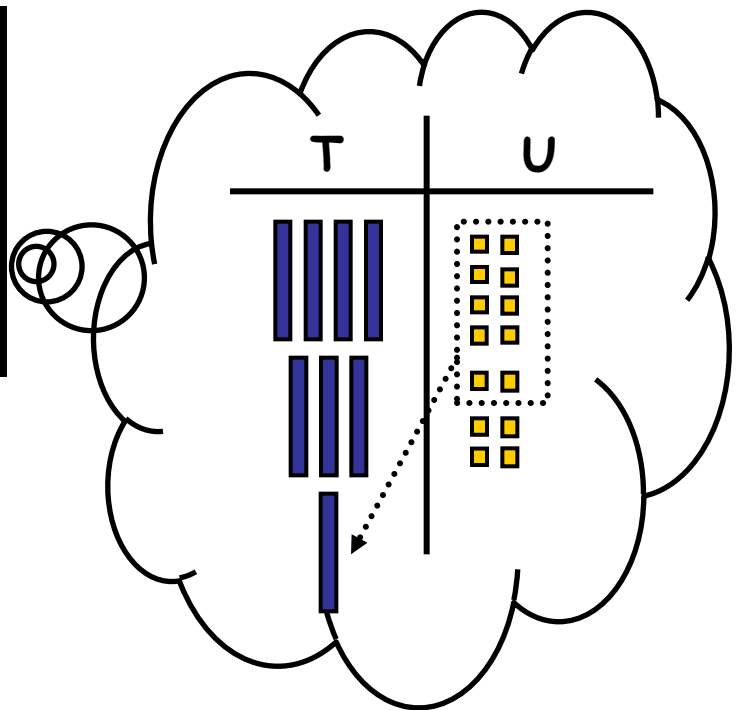
$$70 + 14 = 84$$

Expanded method

It is important that the children have a good understanding of place value and partitioning using concrete resources and visual images to support calculations. The expanded method enables children to see what happens to numbers in the standard written method.

$$48 + 36$$

$$\begin{array}{r} 48 \\ + \underline{36} \end{array}$$



$$\begin{array}{r} \text{T} \quad \text{U} \\ 4 \quad 8 \\ + \underline{3 \quad 6} \\ 1 \quad 4 \text{ (U)} \\ + \underline{7 \quad 0 \text{ (T)}} \\ 8 \quad 4 \end{array}$$

$$\begin{array}{r} 48 \\ + \underline{36} \\ 84 \\ \hline 1 \end{array}$$

Standard written method

The previous stages reinforce what happens to the numbers when they are added together using more formal written methods.

Older children consolidate this method by using larger numbers and adding decimals.

Progression in Teaching Subtraction

Mental Skills

Recognise the size and position of numbers

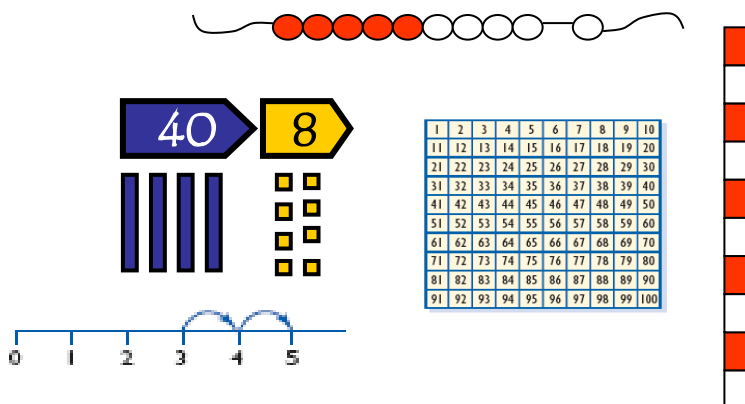
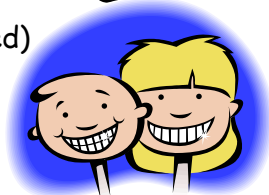
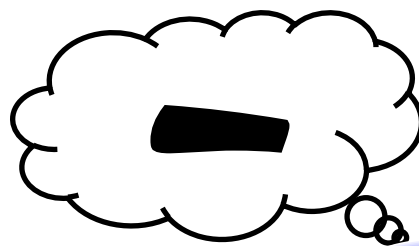
Count back in ones and tens

Know number facts for all numbers to 20

Subtract multiples of 10 from any number

Partition and recombine numbers (only partition the number to be subtracted)

Bridge through 10



Key Vocabulary

subtract

take away

minus

count back

less


fewer

difference between

count back take away
fewer subtract
minus less
difference between

Begin to count backwards in familiar contexts such as number rhymes or stories

Five fat sausages frying in a pan ...



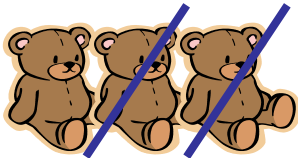
Ten green bottles hanging on the wall ...



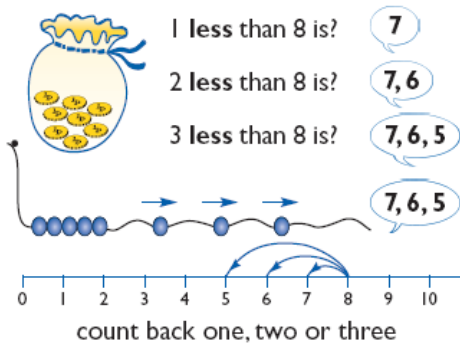
10, 9, 8, 7, ...

Continue the count back in ones from any given number

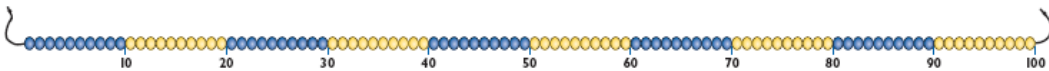
Begin to relate subtraction to 'taking away'



Three teddies **take away** two teddies leaves one teddy



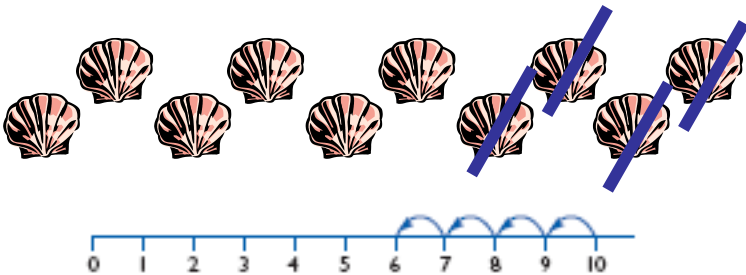
Find one less than a number



Count back in tens



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



If I **take away** four shells there are six left

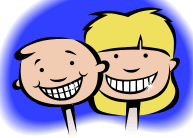
Count backwards along a number line to 'take away'

Begin to use the - and = signs to record mental calculations in a number sentence

Maria had six sweets and she ate four. How many did she have left?



$$6 - 4 = 2$$



$$6 + ? = 10$$

$$10 - 6 = ?$$

$$? + 6 = 10$$

$$10 - 4 = 6$$



$$20 = 12 + 8$$

$$8 + 12 = 20$$

$$20 - 8 = 12$$

$$20 - 12 = 8$$

Know by heart subtraction facts for numbers up to 10 and 20

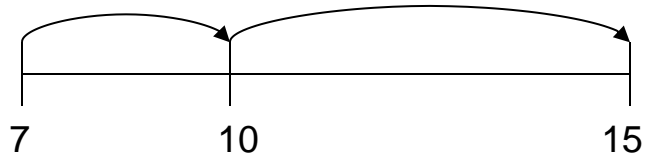
'Counting on'

Subtract single digit numbers often bridging through 10

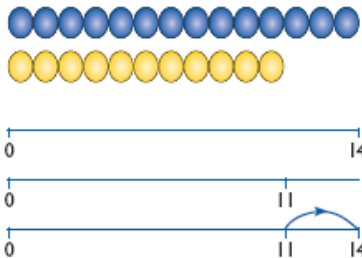
+3

+5

$$15 - 7 = 8$$



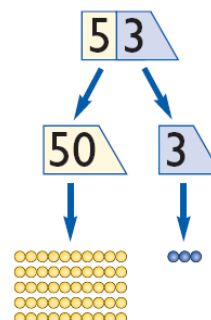
The difference is?



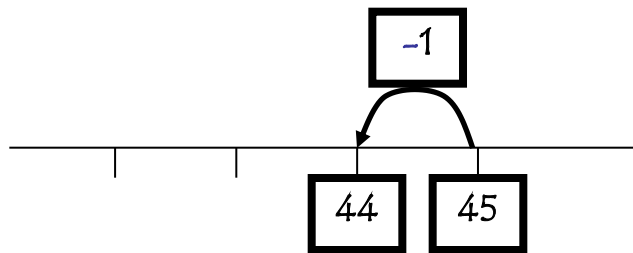
The difference between 11 and 14 is 3.
 $14 - 11 = 3$
 $11 + \square = 14$

Begin to find the difference by counting up from the smallest number

Begin to partition numbers in order to take away



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



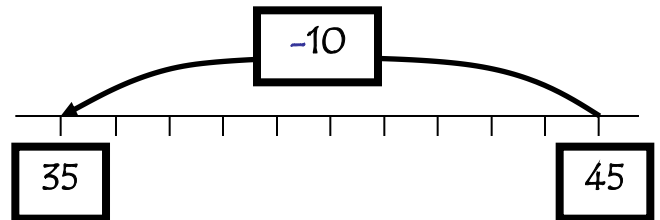
Subtract 1 from a two-digit number

$$45 - 1$$

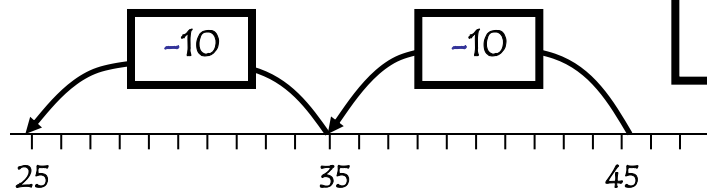
Subtract 10 from a two-digit number

$$45 - 10$$

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



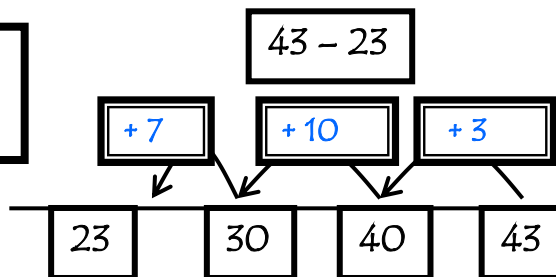
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



Subtract multiples of 10 from any number

$$45 - 20$$

Partition the number to be subtracted (no exchanging)



$$43 - 23$$

$$+7$$

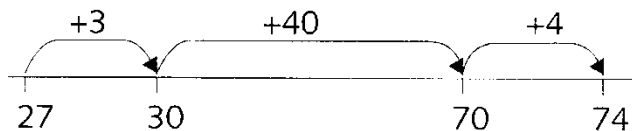
$$+10$$

$$+3$$

$$43 - 20 = 23$$

$$23 - 3 = 20$$

Decide whether to count on or count back



$$74 - 27 = 47$$

Now what's the answer?

Partitioning number to be subtracted - with exchanging (links to counting back on number line)

$$43 - 27 = 16$$

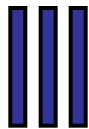


$$43 - 20 = 23$$

$$23 - 7 = 16$$

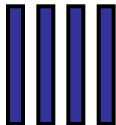

$$23 - 7 = 16$$

$$43 - 27 = 16$$



to subtract 7 units
we need to exchange
a ten for ten units



T	U
	
- 2	7

Expanded method

It is important that the children have a good understanding of place value and partitioning using concrete resources and visual images to support calculations. The expanded method enables children to see what happens to numbers in the standard written method.

$$\begin{array}{r} 30 \quad \cancel{40} \\ - 20 \\ \hline 10 \end{array} \quad \begin{array}{r} + 10 + 3 \\ + 7 \\ \hline + 6 \end{array}$$

Consolidate this method with larger numbers

Standard written method

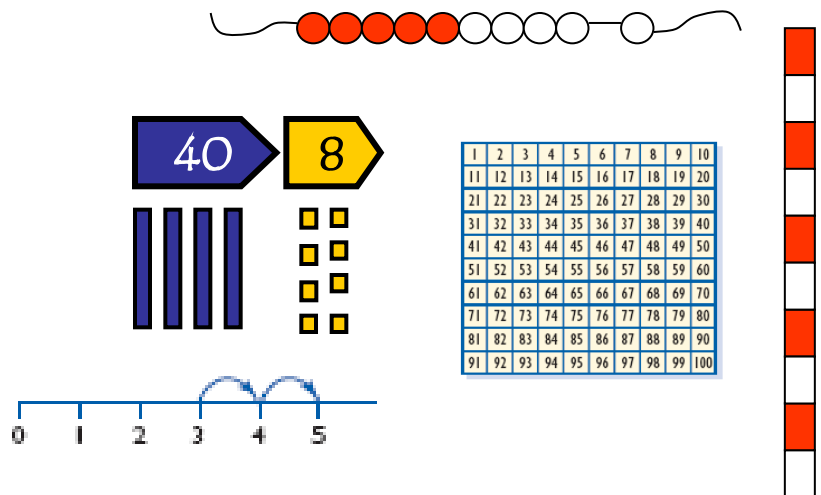
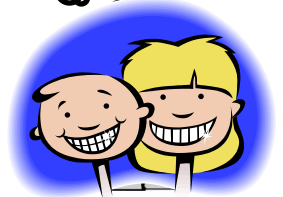
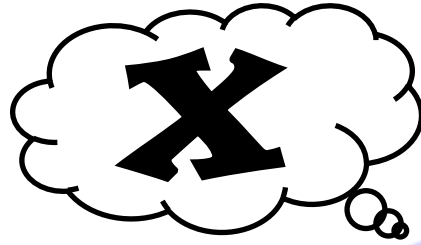
The previous stages reinforce what happens to numbers when they are subtracted using more formal written methods. It is important that the children have a good understanding of place value and partitioning.

$$\begin{array}{r} 3 \quad \cancel{4} \quad 13 \\ - 27 \\ \hline 16 \end{array}$$

Progression in Teaching Multiplication

Mental Skills

Recognise the size and position of numbers
Count on in different steps 2s, 5s, 10s
Double numbers up to 10
Recognise multiplication as repeated addition
Quick recall of multiplication facts
Use known facts to derive associated facts
Multiplying by 10, 100, 1000 and understanding the effect
Multiplying by multiples of 10

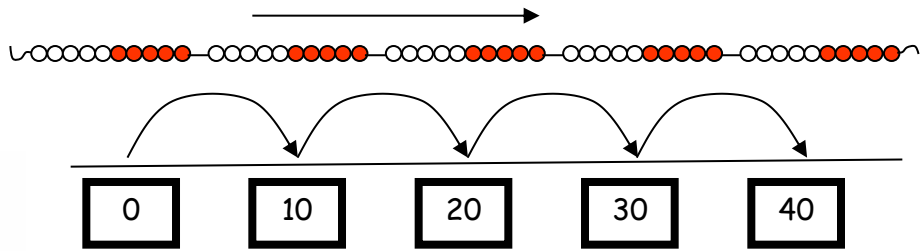


Vocabulary

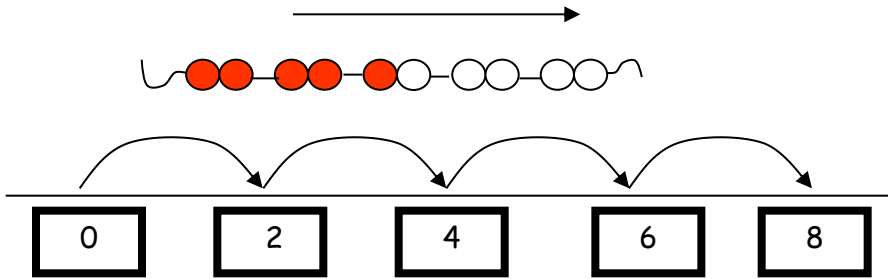
lots of
groups of
times
multiply
multiplication
multiple
product
once, twice, three times
array, row, column
double
repeated addition

multiplication product
once, twice, three times
double groups of
repeated addition lots of
array, row, column multiply
times multiple

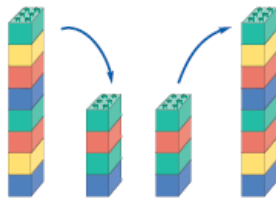
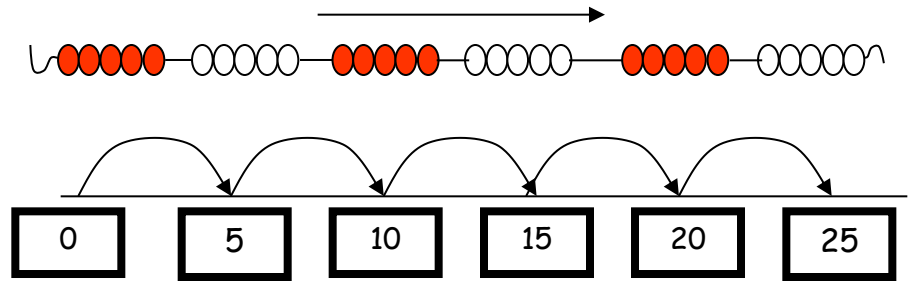
Count in tens
from zero



Count in twos
from zero



Count in fives
from zero



half of 8 is 4
 $8 \div 2 = 4$

double 4 is 8
 $4 \times 2 = 8$

Know doubles and
corresponding halves

Know multiplication tables to 10×10

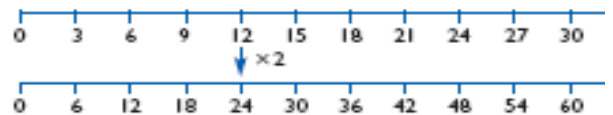
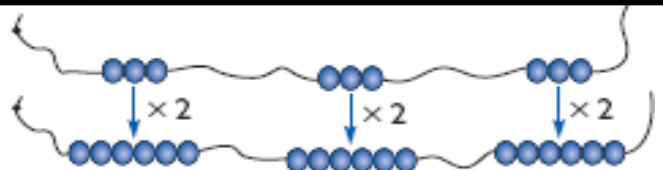
$$2 \times 5 = 10$$

$$\times 5$$

$$6 \times 5 = 30$$

$$3 \times 5 = 15$$

$$8 \times 5 = 40$$



$$12 \times 2 = 24$$

Twice as
many

Use known facts to
work out new ones



$$2 + 2 + 2 + 2$$

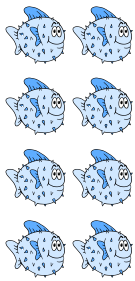
Understand multiplication
as repeated addition

$$2 + 2 + 2 + 2 = 8$$

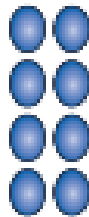
$$4 \times 2 = 8$$

2 multiplied by 4

4 lots of 2

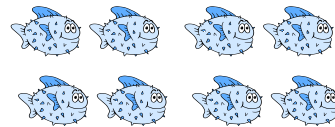


$$2 \times 4$$



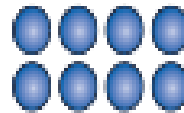
$$2 \times 4 = 8$$

$$4 \times 2 = 8$$



$$4 \times 2$$

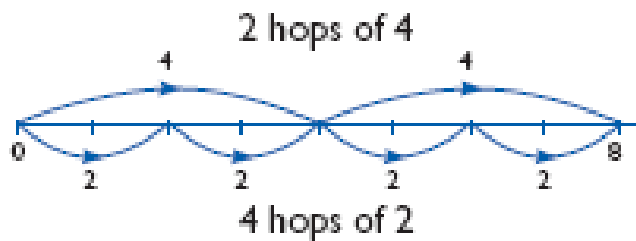
Understand
multiplication
as an array



$$4 \times 2 = 8$$

$$2 \times 4 = 8$$

Understand how to
represent arrays
on a number line



Understand that ...

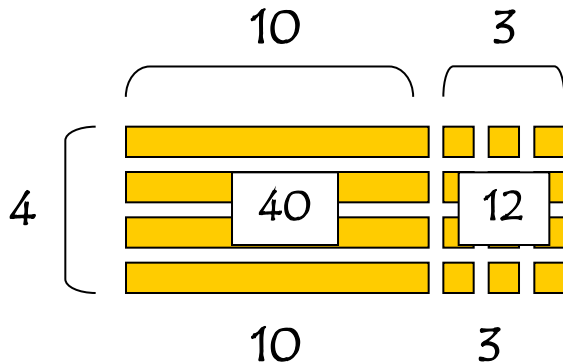
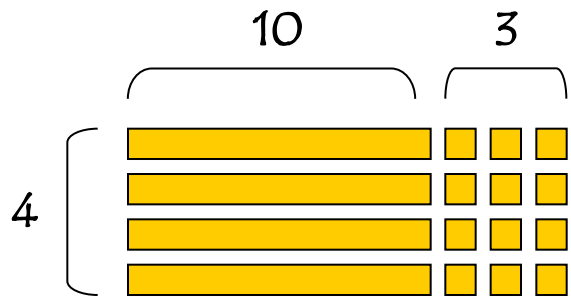
$$24 \times 20 = 24 \times 2 \times 10$$

$$24 \times 50 = 24 \times 5 \times 10$$

Use factors to multiply

Use place value apparatus to support the multiplication of $U \times TU$

$$4 \times 13$$



Use place value apparatus to support the multiplication of $U \times TU$ alongside the grid method

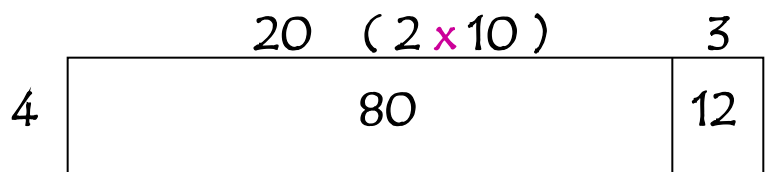
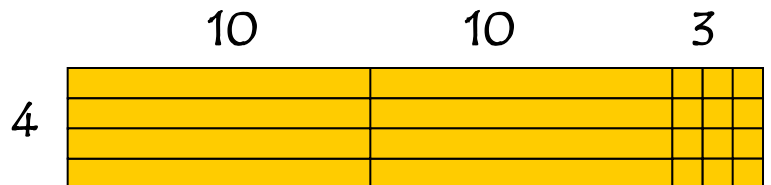
$$4 \times 13$$



$$40 + 12 = 52$$

Use place value apparatus to represent the multiplication of $U \times TU$ alongside the grid method

$$4 \times 23$$



$$80 + 12 = 92$$

Multiplying TU x TU

$$14 \times 33$$

	30	3	
10	300	30	= 330 +
4	120	12	= 132
			<u>462</u>

300
120
30
+ 12
<u>462</u>

56	
\times 27	
392	(56 \times 7)
<u>1120</u>	(56 \times 20)
1512	
1	

Standard written method

Progression in Teaching Division

Mental Skills

Recognise the size and position of numbers

Count back in different steps 2s, 5s, 10s

Halve numbers to 20

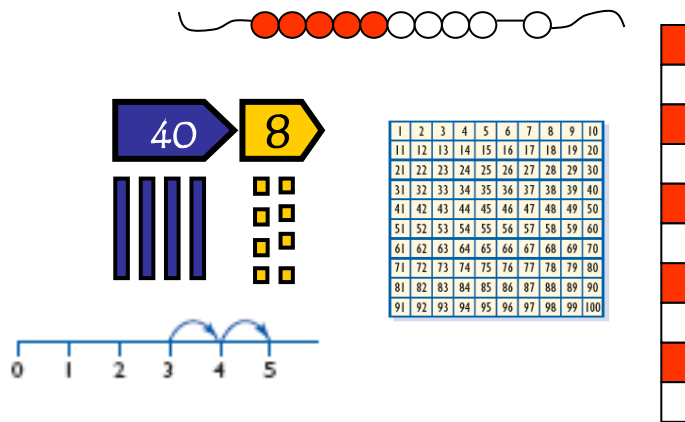
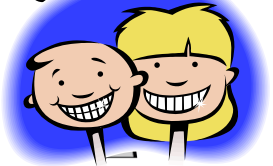
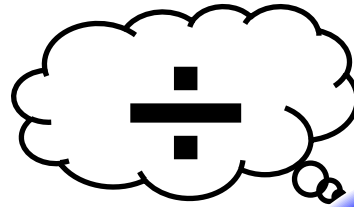
Recognise division as repeated subtraction

Quick recall of division facts

Use known facts to derive associated facts

Divide by 10, 100, 1000 and understanding the effect

Divide by multiples of 10



Vocabulary

lots of

groups of

share

group

halve

half

divide

division

divided by

remainder

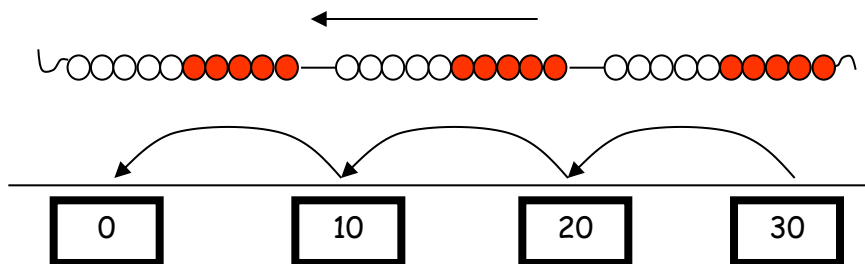
factor

quotient

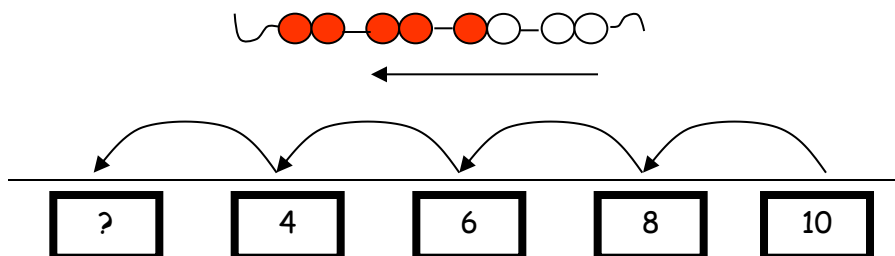
divisible

group groups of
lots of divide
divided by quotient
division factor
remainder divisible
half halve share

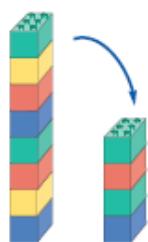
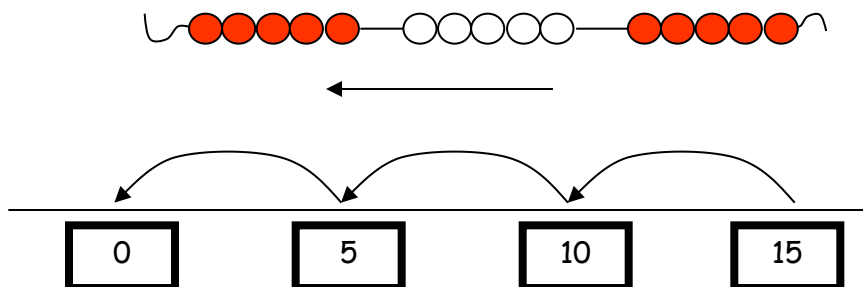
Count back in tens



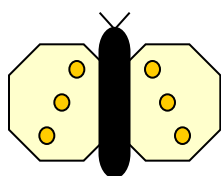
Count back in twos



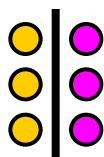
Count back in fives



half of 8 is 4
 $8 \div 2 = 4$



Half of 6 is 3
 $\frac{1}{2}$ of 6 = 3



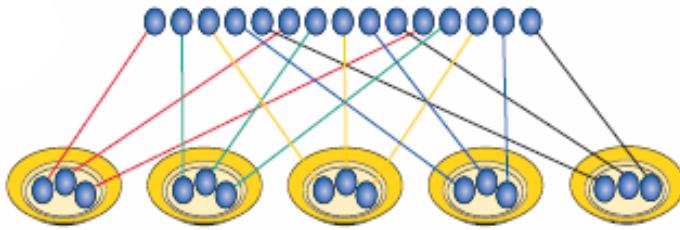
Know halves

Use known multiplication facts to work out corresponding division facts

If $2 \times 10 = 20$
then
 $20 \div 10 = 2$
 $20 \div 2 = 10$

$$15 \div 3 = 5$$

15 shared between 5



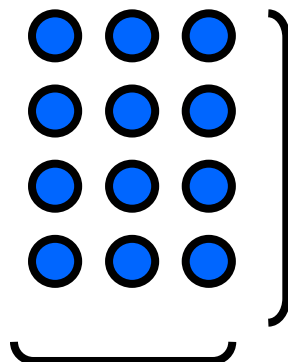
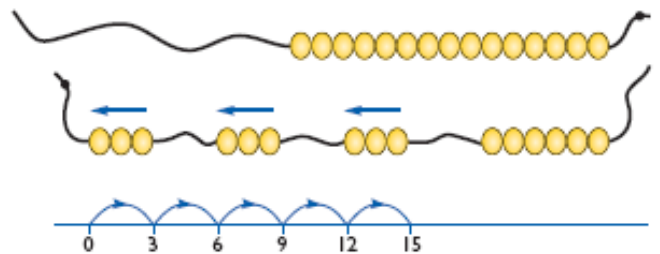
Understand division
as sharing

Understand division
as grouping

How many 3s
in 15?



$$15 \div 3 = 5$$



12 divided into groups
of 3 gives 4 groups

$$12 \div 3 = 4$$

12 divided into groups
of 4 gives 3 groups

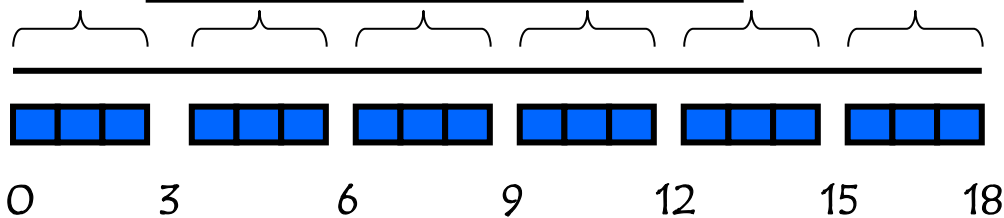
$$12 \div 4 = 3$$

Reinforce division as
grouping through the
use of arrays

Represent 'groups' for division on a number line using apparatus alongside the line

18 divided into groups of 3

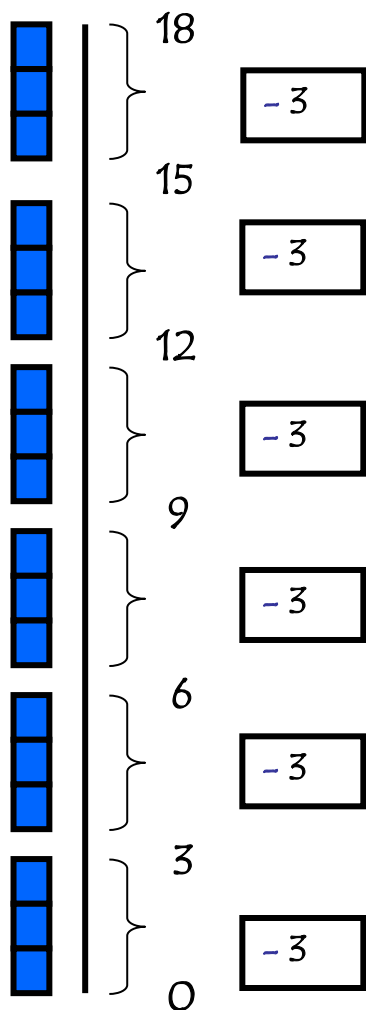
$$18 \div 3 = 6$$



$$18 \div 3 = 6$$



$$18 \div 6 = 3$$



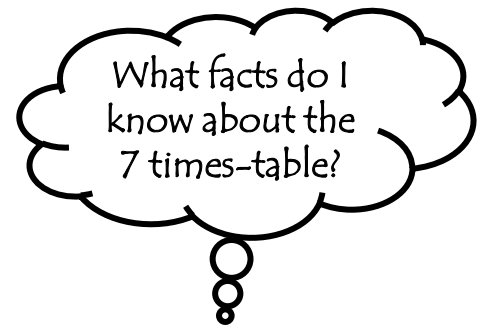
$$18 \div 3 = 6$$

$$\begin{array}{r} 18 \\ - 3 (1 \times 3) \\ \hline 15 \\ - 3 (1 \times 3) \\ \hline 12 \\ - 3 (1 \times 3) \\ \hline 9 \\ - 3 (1 \times 3) \\ \hline 6 \\ - 3 (1 \times 3) \\ \hline 3 \\ - 3 (1 \times 3) \\ \hline 0 \end{array}$$

Understand division as repeated subtraction using a vertical line and apparatus to make the links

Year 5 look at repeated subtraction of larger chunks

Children need to see that as the numbers get larger, large chunk subtraction is the more efficient method. Multiples of the divisor (large chunks) are taken away. Multiplication facts are needed to see the size of the 'chunk'.



$$100 \div 7 = 14 \text{ r } 2$$

$$\begin{array}{r} 100 \\ - 70 \quad (10 \times 7) \\ \hline 30 \\ - 28 \quad (4 \times 7) \\ \hline 2 \end{array}$$

$$518 \div 7 = 74$$

$$\begin{array}{r} 518 \\ - 350 \quad (50 \times 7) \\ \hline 168 \\ - 140 \quad (20 \times 7) \\ \hline 28 \\ - 28 \quad (4 \times 7) \\ \hline 0 \end{array}$$

Fact Box

$$1 \times 7 = 7$$

$$2 \times 7 = 14$$

$$5 \times 7 = 35$$

$$10 \times 7 = 70$$

$$20 \times 7 = 140$$

$$50 \times 7 = 350$$

$$100 \times 7 = 700$$

$$560 \div 24$$

$$\begin{array}{r} 23 \text{ r } 8 \\ 24 \overline{) 560} \\ - 480 \quad (20 \times 24) \\ \hline 80 \\ - 72 \quad (3 \times 24) \\ \hline 8 \end{array}$$

Standard written method

Links directly to large chunk subtraction